Maths Calculation Policy: Stages* for Formal Written Methods from EYFS to Year 6

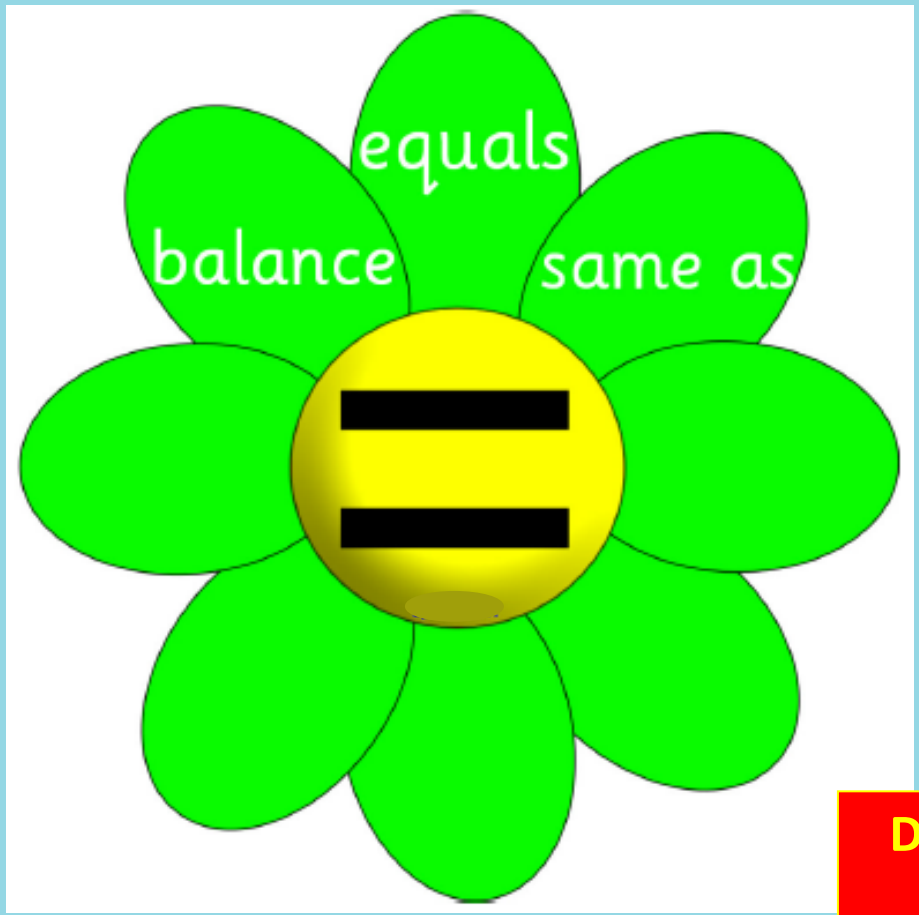
*Note: stage number does not equate to year group (see page 2)

Overview of suggested stages to reach by end of year for on-track+ pupils in each year group:

Operation	+ addition	- subtraction	x multiplication	÷ division
EYFS	1	1	1	1
Year 1	3b	3b	2b	3
Year 2	4a	5a	3	4
Year 3	5	6	4	5
Year 4	6	7	5	6a
Year 5	Consolidate	Consolidate	6	6b
Year 6	Consolidate	Consolidate	Consolidate	7

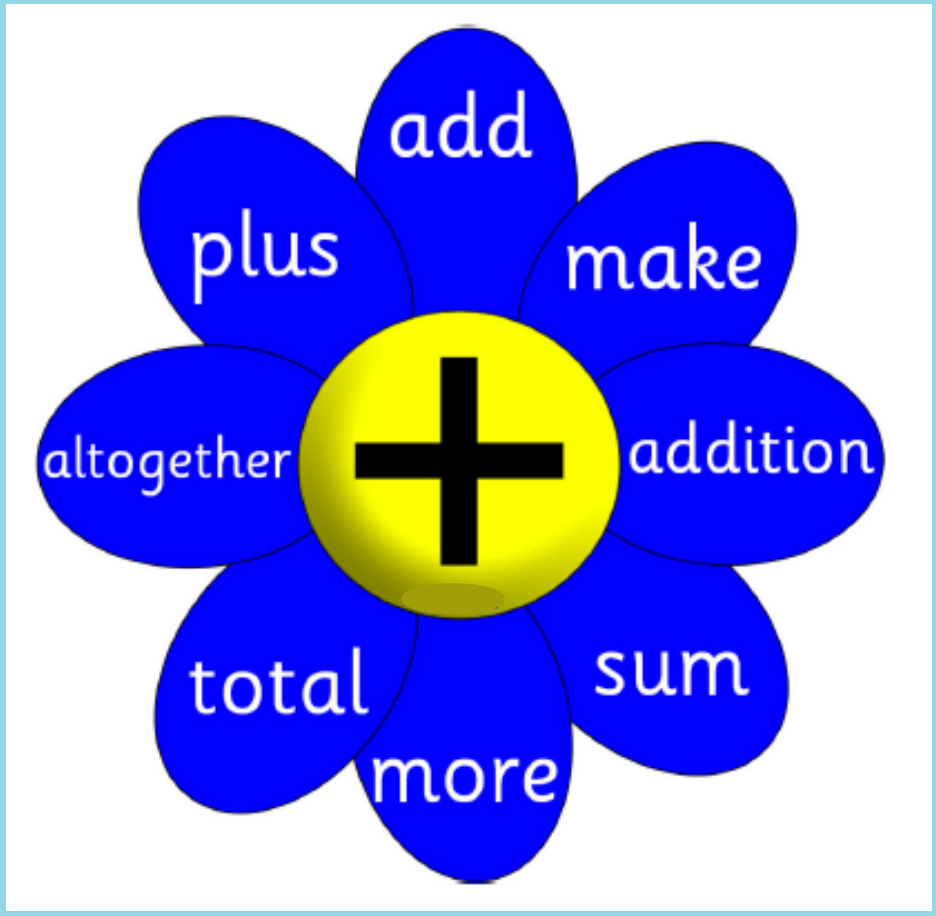
<u>Vocabulary</u>: Pupils need to constantly hear a variety of calculation vocabulary, particularly for missing number problems

Vocabulary for the ‘equals’ sign:



Does NOT mean ‘the answer’

Vocabulary for addition operation:

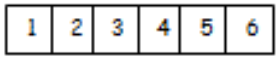


as well as

additional

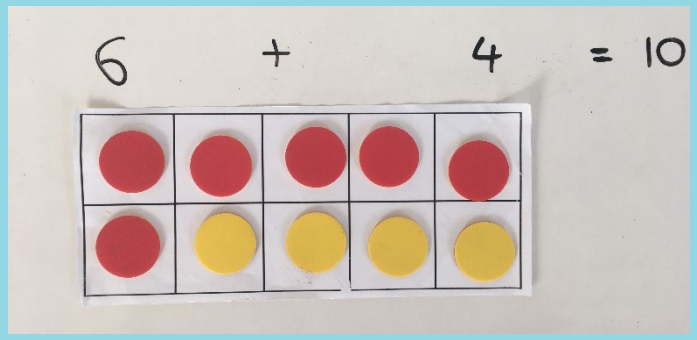
extra

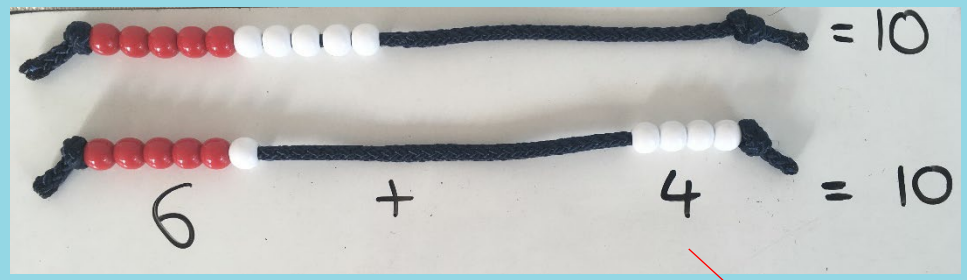
Stage 1: Addition as early counting forwards to 10 and then 20, using concrete resources (fingers, everyday objects, counting bears, Numicon etc.)

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
If available, Numicon shapes are introduced straight away and can be used to: • identify 1 more/less • combine pieces to add. • find number bonds. • add without counting.  Children can record this by printing or drawing around Numicon pieces. Children begin to combine groups of objects using concrete apparatus  Construct number sentences verbally or using cards to go with practical activities. Children are encouraged to read number sentences aloud in different ways "Three add two equals 5" "5 is equal to three and two" Children make a record in pictures, words or symbols of addition activities already carried out. Solve simple problems using fingers  5 + 1 = 6 Number tracks can be introduced to count up on and to find one more:  What is 1 more than 4? 1 more than 13?	**Games and songs can be a useful way to begin using vocabulary involved in addition e.g.** Alice the Camel add more and make sum total altogether score double one more, two more, ten more…

Stage 2a: Addition as using number bond facts up to 20 (and related facts to 100 in Y2)

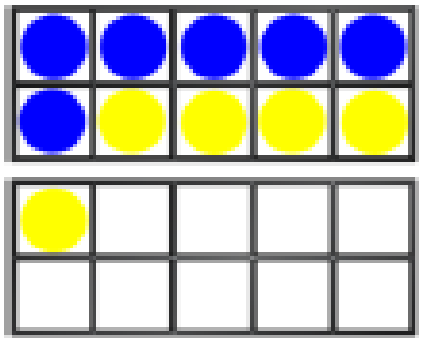
Concrete representation: e.g. tens frames, bead strings, Numicon, hundred squares, etc.





Abstract representation alongside

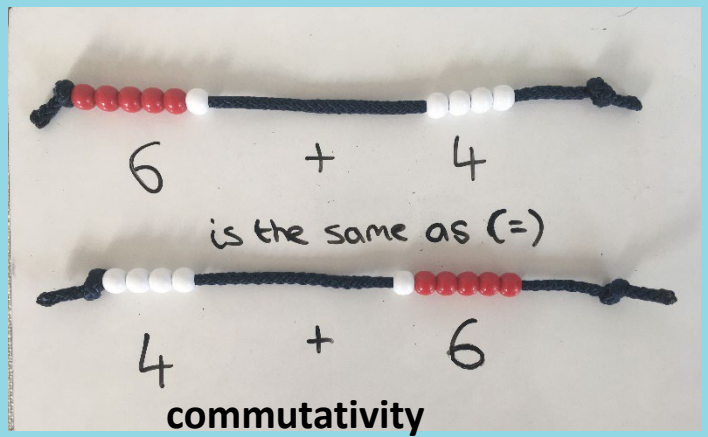
Pictorial representation: e.g. tens frames



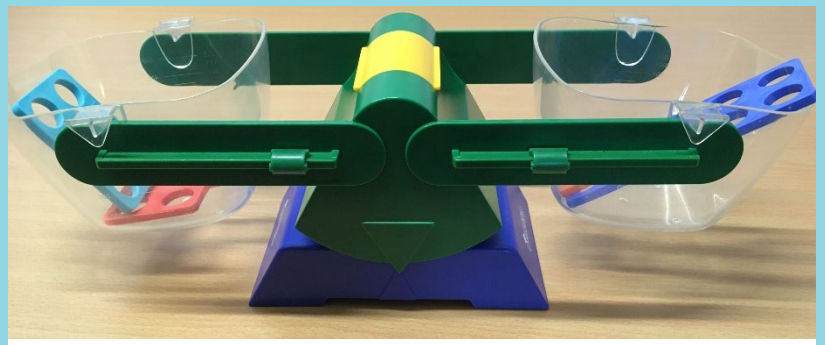
6 + 5 = 11

Stage 2b: Addition as developing an understanding of inverse, commutativity and balancing

Concrete representation:



commutativity



6 + 5 = 10 + 1

Numicon is weighted, so it will 'balance'

Abstract representation:

11 - 6 = 5

$6 + \square = 11$

$6 + 5 = 5 + \square$

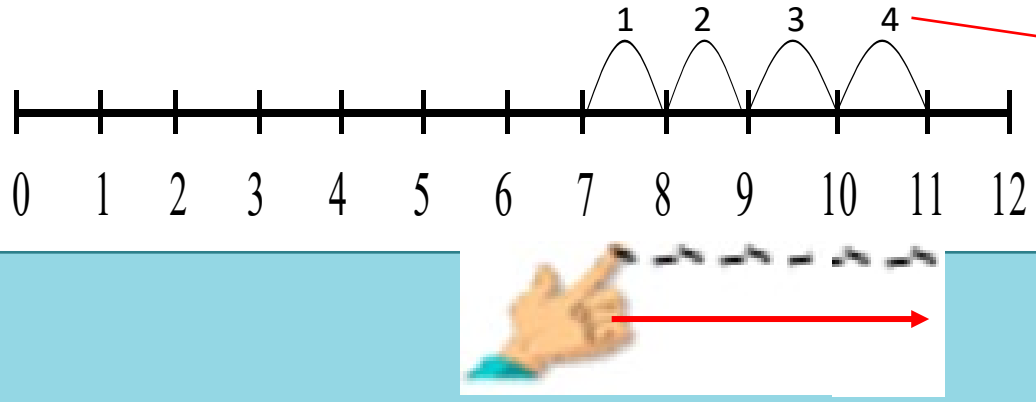
$6 + 5 = \square + 4$

6 + 5 = 5 + 6

6 + 5 = 7 + 4

Stage 3a: Addition as counting on, on a filled number line

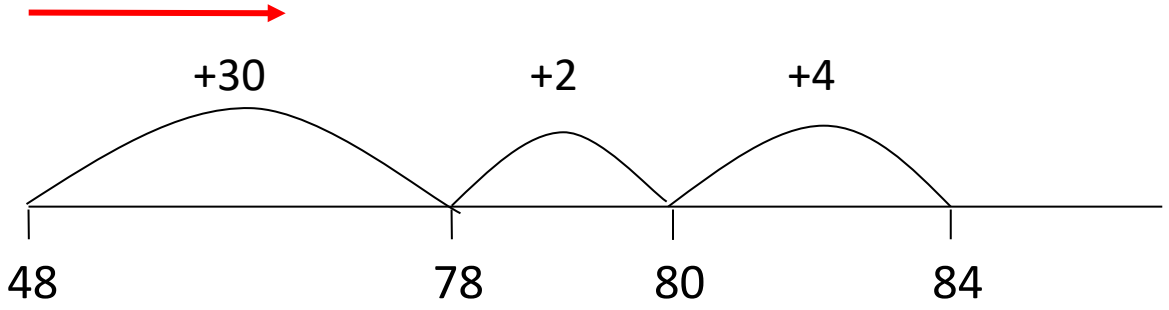
7 + 4 = 11



Check counting by labelling steps afterwards

Stage 3b: addition as counting on in 10s and 1s, on an empty number line

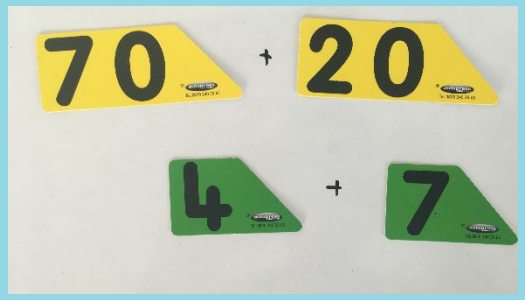
48 + 36 = 84



Stage 3c (also a mental method): Addition by partitioning both numbers, before adding

74 + 27 = 101

Concrete:



Abstract:

74 + 27

70 + 20 = 90

4 + 7 = 11

90 + 11 = 101

* A more efficient way, when secure in this concept, is to only partition the second number before adding:

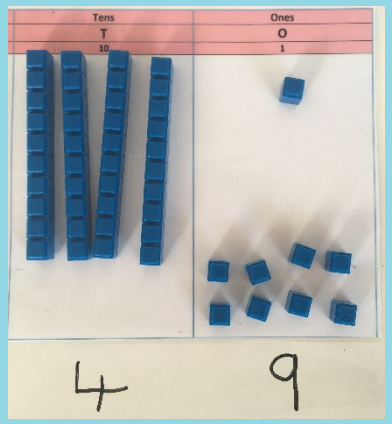
74 + 20= 94

94 + 7 = 101

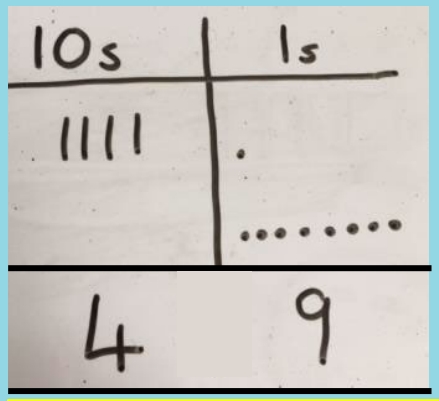
Stage 4a: Base 10 column method (no exchanging)

41 + 8 = 49

Concrete:



Pictorial:



Encourage pupils to draw the place value chart along gridlines neatly, without a ruler, and to draw the 10 'sticks' and 1 'dots' fitting into one grid square each.

Steps (with language to use):

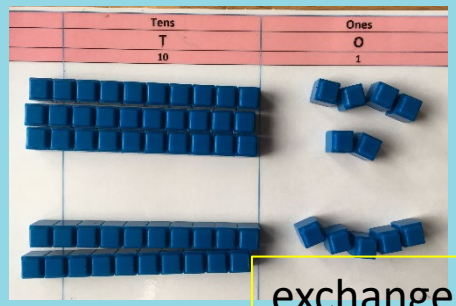
1. Use/Draw a place value chart (label 10s, 1s).
2. Put down/Draw the tens and ones base 10 from the calculation.
3. Add the ones first, from both numbers. Write the total number of ones altogether underneath.
4. Add the tens next, from both numbers. Write the total number of tens altogether underneath.

Stage 4b: Base 10 column method (with exchanging)

36 + 25 = 61

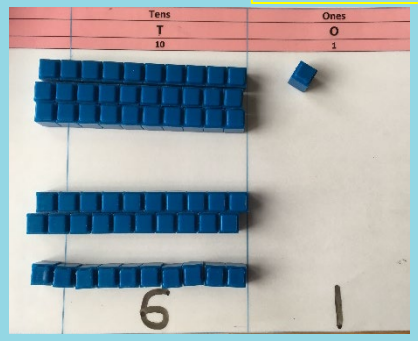
Concrete:

1:

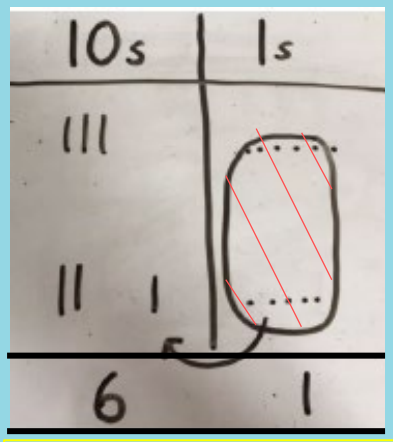


exchange

2:



Pictorial:



Encourage pupils to draw the place value chart along gridlines neatly, without a ruler, and to draw the 10 'sticks' and 1 'dots' fitting into one grid square each.

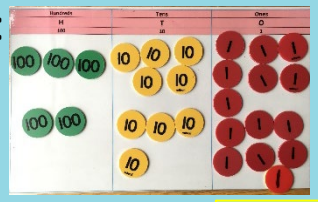
Steps (with language to use):

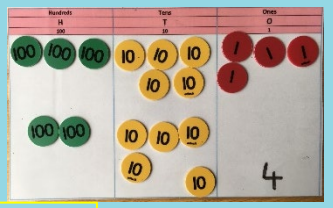
1. Use/Draw a place value chart (label 10s, 1s).
2. Put down/Draw the tens and ones base 10 from the calculation.
3. Add the ones column first (if there are more than 9, EXCHANGE 10 ones for 1 ten, cross the ones out).
4. Write the total of ones left, underneath.
5. Add the tens next. Write the total of tens, underneath.

Stage 5: Place Value Counters column method (draw compact column method alongside when more secure)

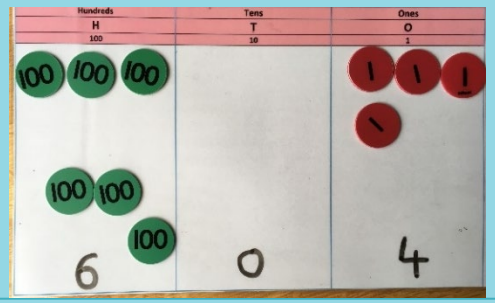
357 + 247 = 604

Concrete:

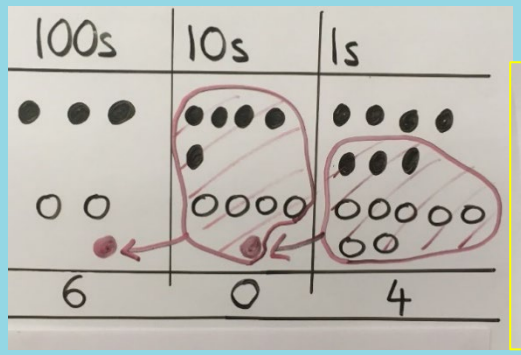
1: 

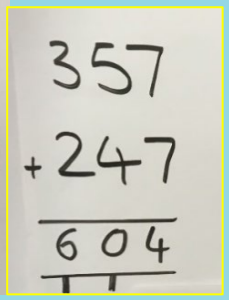
2: 

exchanging

3: 

Pictorial:





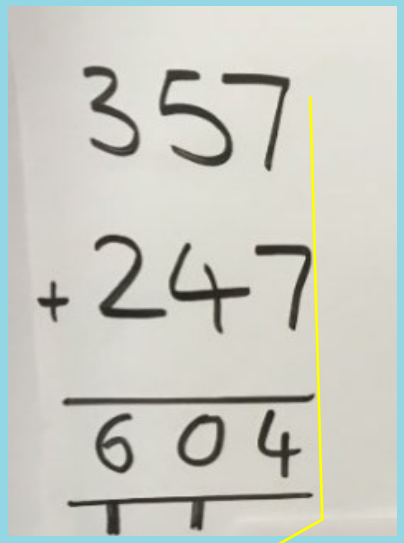
Steps (with language to use):

1. Draw a place value chart (label HTO).
2. Draw H,T,O counters from the calculation.
3. Add the ones column first (if there are more than 9, EXCHANGE 10 ones for 1 ten, cross them out).
4. Write the total of ones left between the equals sign, at the bottom.
5. Add the tens next (if there are more than 9, EXCHANGE 10 tens for 1 hundred, cross them out).
6. Write the total of tens left between the equals sign, at the bottom.
7. Add the hundreds next.
8. Write the total of hundreds left between the equals sign, at the bottom.

Stage 6: Short column method

Abstract:

357 + 247 = 604



7 + 7 = 14 so the 4 ones go in the ones column and we exchange the other 10 ones for 1 ten underneath the tens column, etc.

Steps (with language to use):

1. Write the digits carefully, lined up in the correct columns.

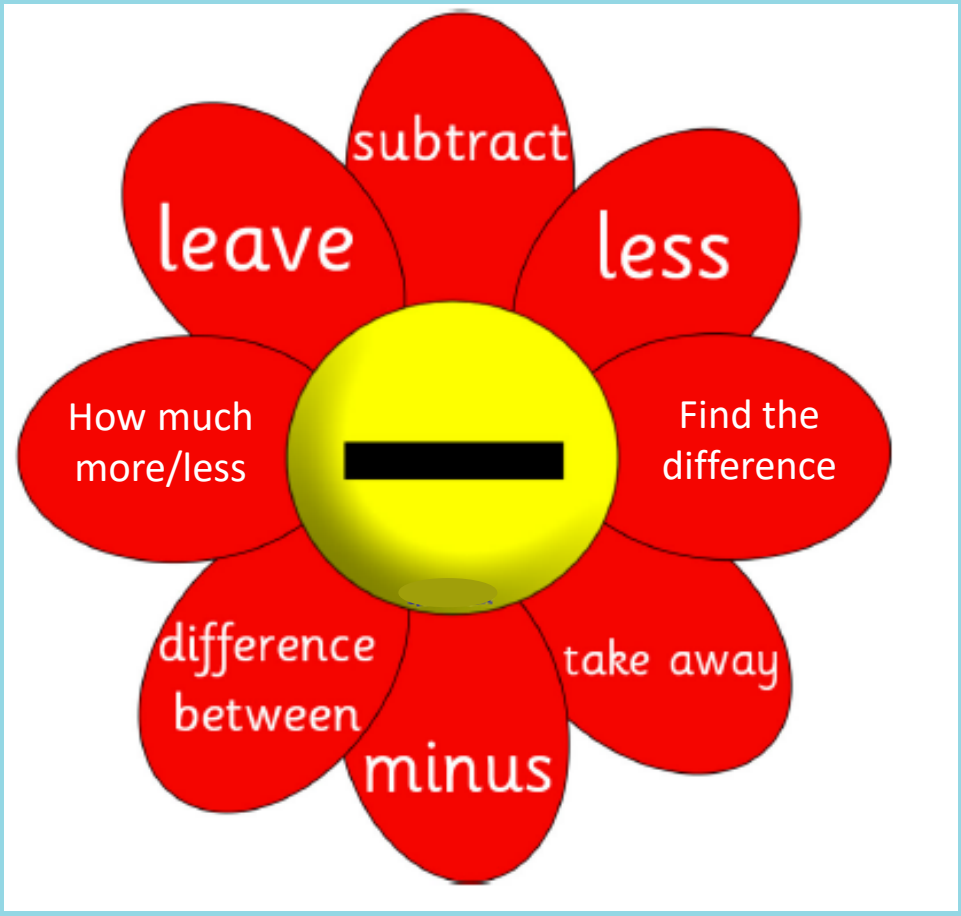
2. Draw the addition and equal to symbols.

3. Add the ones column first.
   Exchange 10 ones for 1 ten underneath the tens column if you need to.

4. Add the tens column.
   Exchange 10 tens for 1 hundred underneath the hundreds column if you need to.

5. Add the hundreds column.

Vocabulary for subtraction operation:



left

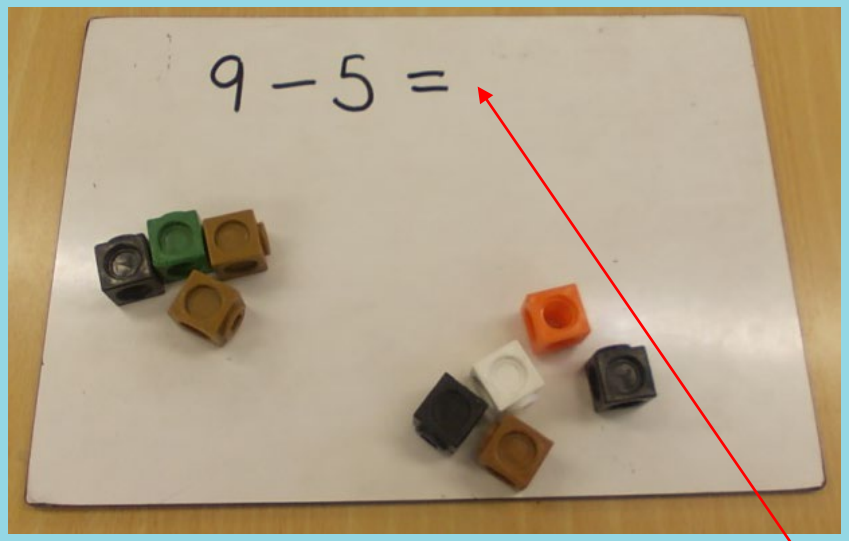
reduction / reduced

fewer

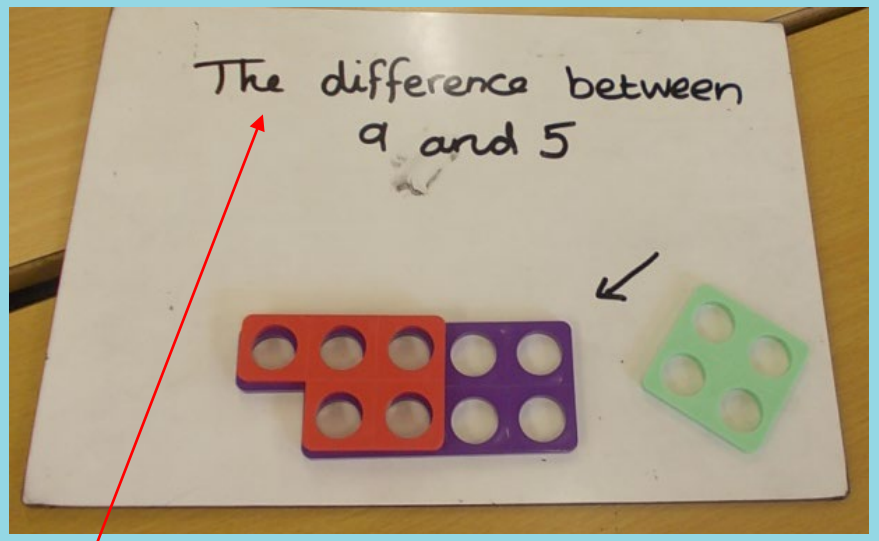
Stage 1: Subtraction as early counting backwards from any point within 20, using concrete resources (fingers, everyday objects, counting bears, Numicon etc.)

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
Children begin with mostly pictorial representations XXX [X X] Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left. Concrete apparatus models the subtraction of 2 objects from a set of 5. Construct number sentences verbally or using cards to go with practical activities. Children are encouraged to read number sentences aloud in different ways "five subtract one leaves four" "four is equal to five subtract one" Children make a record in pictures, words or symbols of subtraction activities already carried out. Solve simple problems using fingers  5 - 1 = 4 Number tracks can be introduced to count back and to find one less: [1][2][3][4][5][6] What is 1 less than 9? 1 less than 20?	**Games and songs can be a useful way to begin using vocabulary involved in subtraction e.g.** Five little men in a flying saucer take (away) leave how many are left/left over? how many have gone? one less, two less... ten less... how many fewer is... than...?

Stage 2a: Subtraction as both 'taking away' and 'finding the difference between', using concrete resources (Numicon, tens frames, bead strings, hundred squares, etc.)



"taking away"

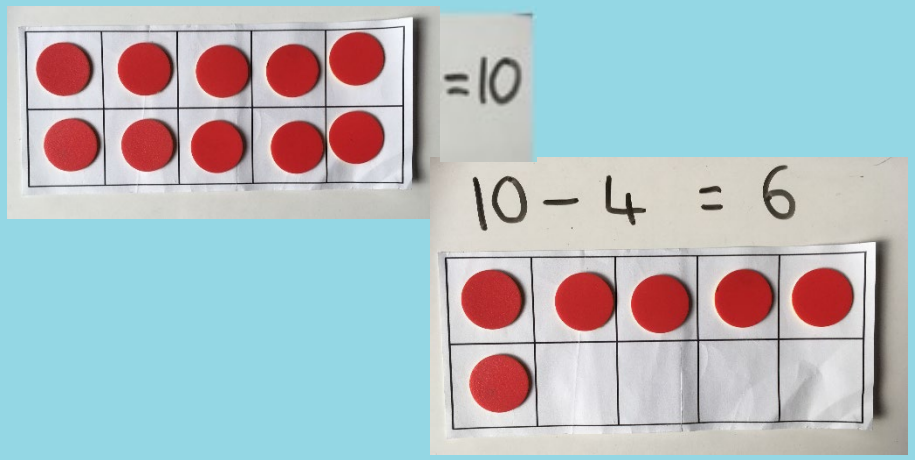


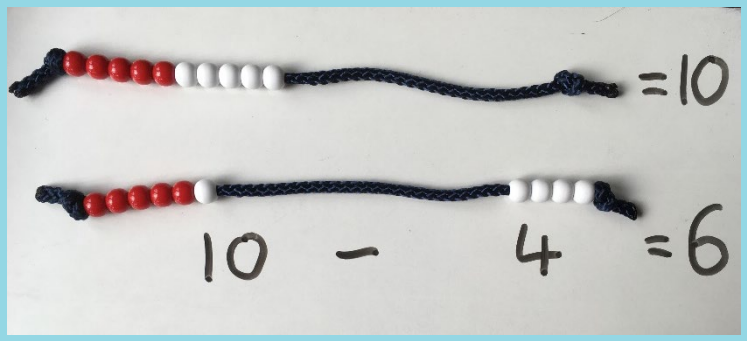
"finding the difference between"

Abstract representation/discussion alongside

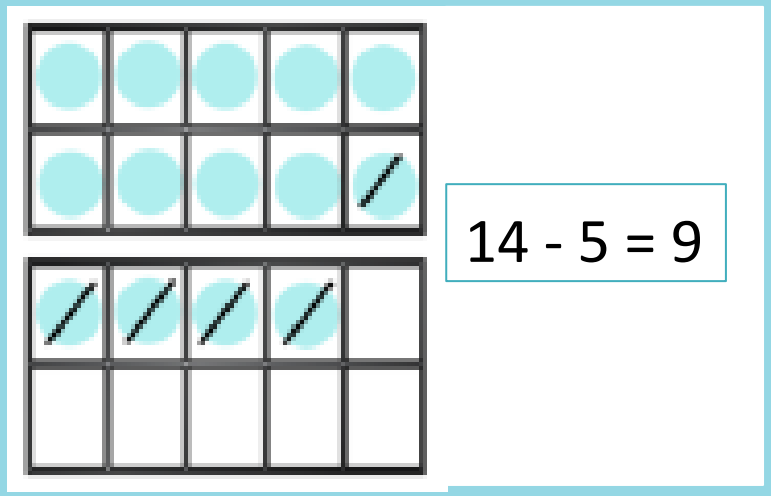
Stage 2b: Subtraction as using number bond facts up to 20 (and related facts to 100 in Y2)

Concrete representation: e.g. tens frames, bead strings, Numicon, hundred squares, etc.





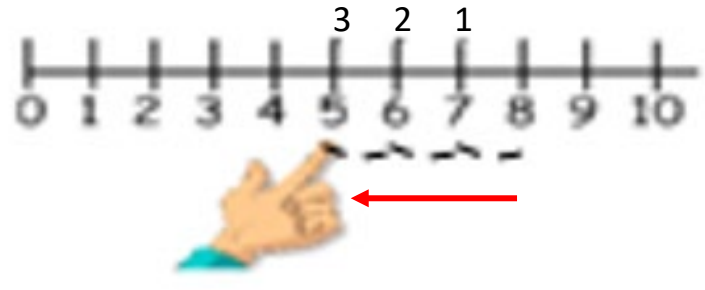
Pictorial representation: e.g. tens frames



* Also, recognise subtraction as the inverse of addition

Stage 3a: Subtraction as 'taking away', by counting backwards on a filled number line

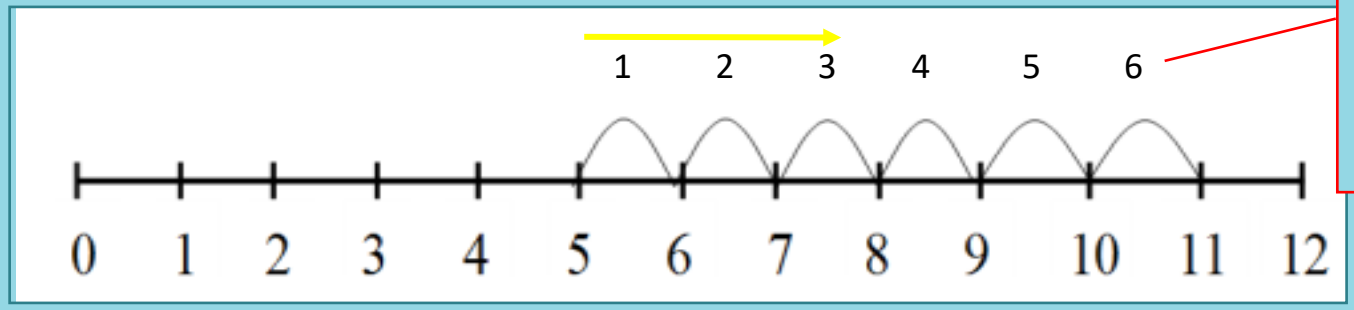
8 – 3 = 5



Check counting by labelling steps afterwards

Stage 3b (also a mental method): Subtraction as 'finding the difference', by counting on, on a filled number line

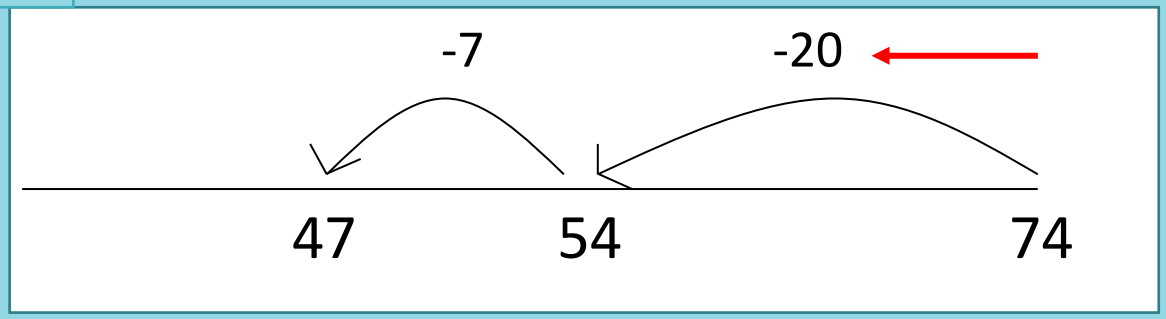
11 - 5 = 6 (means 'the difference between 11 and 5 is 6')



Check counting by labelling steps afterwards

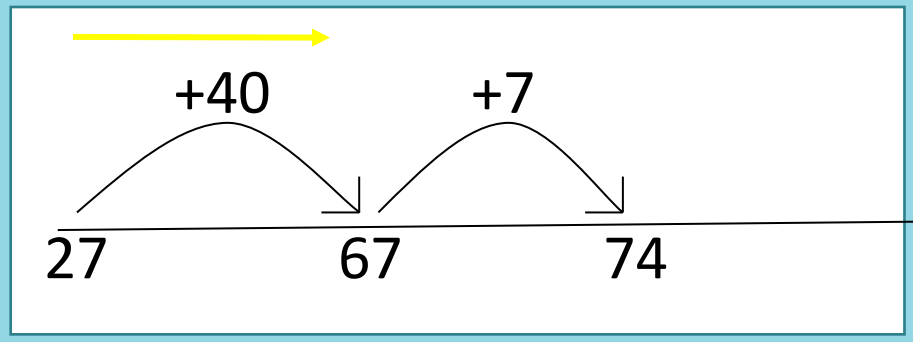
Stage 4a: Subtraction as 'taking away', by counting backwards on an empty number line

74 – 27 = 47



Stage 4b (also a mental method): Subtraction as 'finding the difference', by counting on, on an empty number line

74 - 27 = 47



* This is also a useful method to use for solving money change and time problems too (see last three slides)

Stage 4c (also a mental method): Subtraction by partitioning the second number, before subtracting

74 - 27 = 47

Concrete:

Abstract:

74 - 27

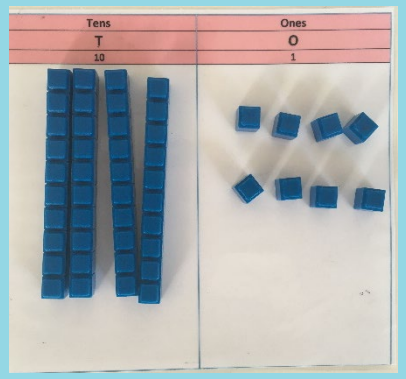
74 – 20 = 54

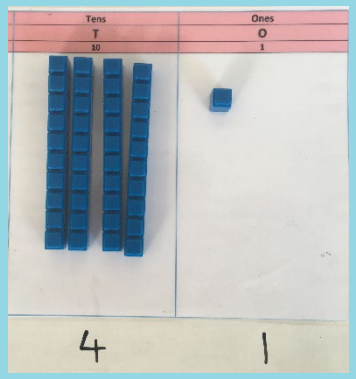
54 - 7 = 47

Stage 5a: Base 10 column method (no exchanging)

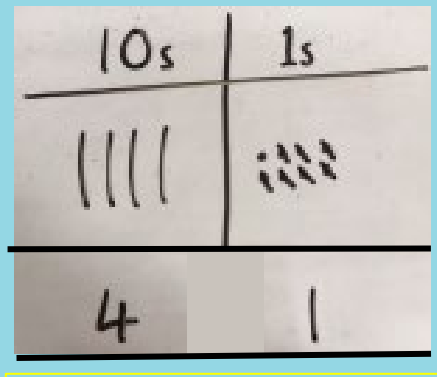
48 - 7 = 41

Concrete:

1: 

2: 

Pictorial:



Encourage pupils to draw the place value chart along gridlines neatly, without a ruler, and to draw the 10 'sticks' and 1 'dots' fitting into one grid square each.

Steps (with language to use):

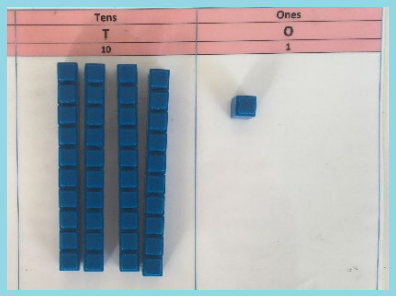
1. Use/Draw a place value chart (label 10s, 1s).
2. Put down/Draw the tens and ones base 10 from the **first number only.**
3. Subtract the ones first, from the second number. Write the total of ones left, underneath.
4. Subtract the tens next, from the second number. Write the total of tens left, underneath.

Stage 5b: Base 10 column method (with exchanging)

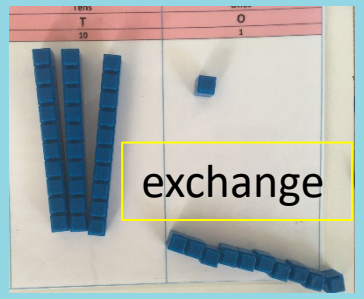
41 - 26 = 15

Concrete:

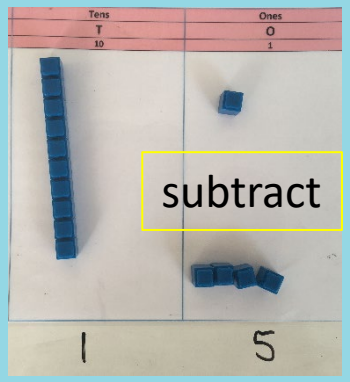
1:



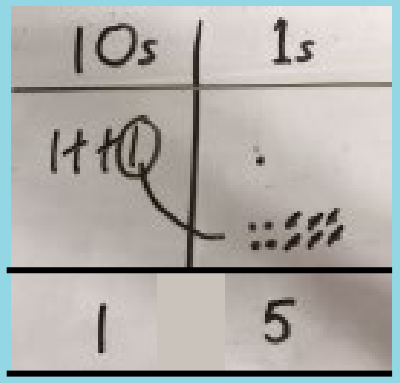
2:



3:



Pictorial:



Encourage pupils to draw the place value chart along gridlines neatly, without a ruler, and to draw the 10 'sticks' and 1 'dots' fitting into one grid square each.

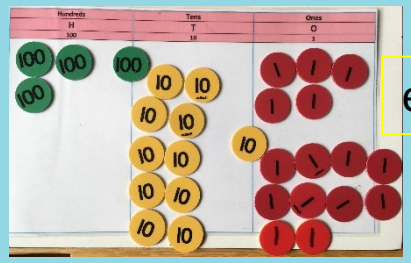
Steps (with language to use):

1. Use/Draw a place value chart (label 10s, 1s).
2. Put down/Draw the tens and ones base 10 from the **first number only.**
3. Subtract the ones first, from the second number (if there are not enough, EXCHANGE one ten for 10 ones, cross the ten out).
4. Write the total of ones left, underneath.
5. Subtract the tens next, from the second number. Write the total of tens left, underneath.

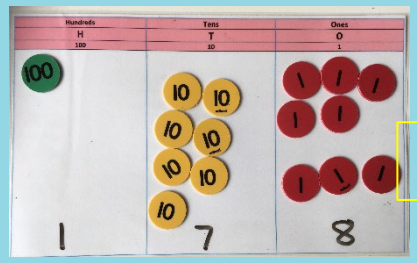
Stage 6: Place Value Counters column method (draw compact column method alongside when more secure)

415 – 237 = 178

Concrete:

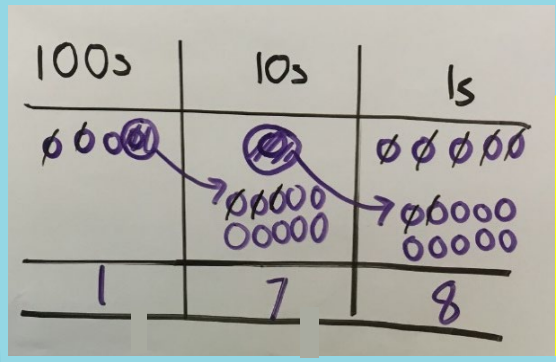


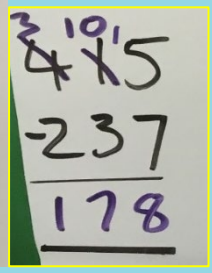
exchanging



subtracting

Pictorial:





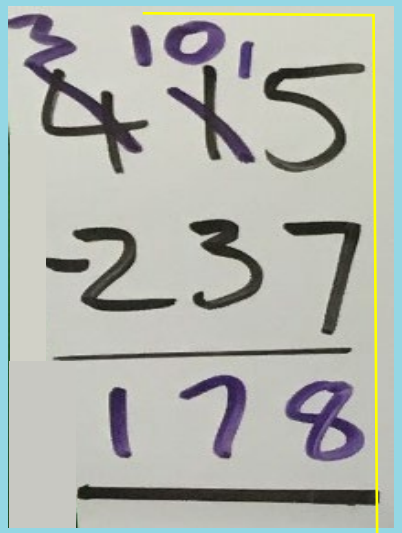
Steps (with language to use):

1. Draw a place value chart (label HTO).
2. Draw H,T,O counters from the first number.
3. Subtract the ones first, from the second number (if there are not enough, EXCHANGE one ten for 10 ones, cross it out) .
4. Write the total of ones between the equals sign, at the bottom.
5. Subtract the tens next (if there are not enough, EXCHANGE one hundred for 10 tens, cross it out).
6. Write the total of tens left between the equals sign, at the bottom.
7. Subtract the hundreds next.
8. Write the total of hundreds left between the equals sign, at the bottom.

Stage 7: Short column method

Abstract:

415 – 237 = 178



5 – 7 would give you a negative number, so exchange 1 ten for 10 ones, so we can now do 15 – 7, etc.

Steps (with language to use):

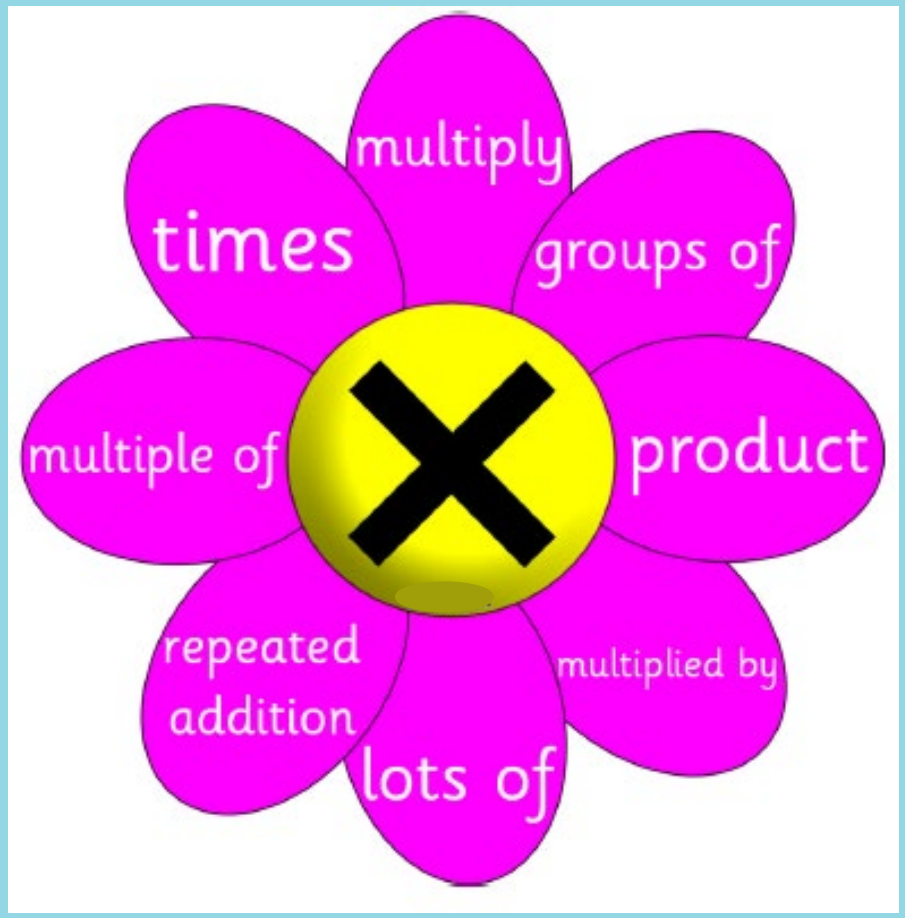
1. Write the digits carefully, lined up in the correct columns.
2. Draw the subtraction and equal to symbols.
3. Subtract the ones column first.
   Exchange (and cross out) 1 ten for 10 ones above the ones column, if you need to.
4. Subtract the tens column.
   Exchange (and cross out) 1 hundred for 10 tens above the tens column, if you need to.
5. Subtract the hundreds column.

3000 – 478

2 1 9 1 9 1
3 0 0 0
- 4 7 8
2 5 2 2

Inefficient method for this calculation – should use a mental method, unless secure at method

Vocabulary for multiplication operation



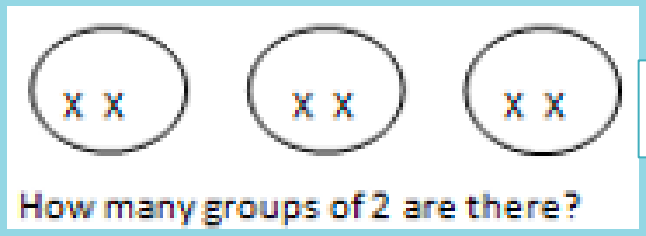
‘n’ times bigger

Stage 1: Multiplication as repeated addition of same sized groups

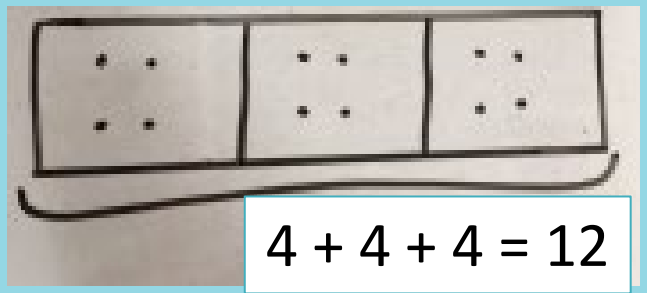
Concrete representation: e.g. everyday objects, counting bears, money, Numicon, etc.

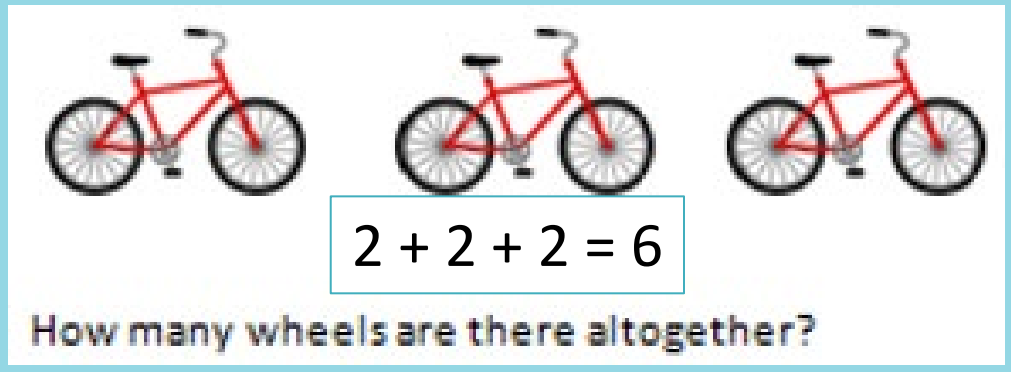
Pictorial representation: e.g. drawings, bar models, etc.

Abstract representation/ discussion alongside



2 + 2 + 2 = 6



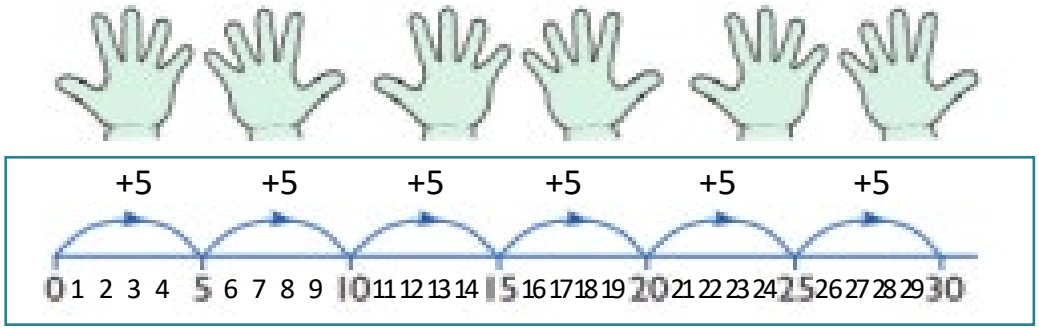


Stage 2a: Multiplication as 'skip counting' on a hundred square

Skip counting by twos									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Year Group	NC Times Tables Expectations
1	**x2 x5 x10** (counting in multiples)
2	**x2 x5 x10** (recall x and ÷ facts)
3	**x3 x4 x8** (recall x and ÷ facts)
4	Recall all x and ÷ facts up to **12 x 12**

Stage 2b: Multiplication as repeated addition, of the same sized groups, on a filled number line



$5 + 5 + 5 + 5 + 5 + 5 = 30$

$5 \times 6 = 30$

5 multiplied by 6

6 groups of 5

Stage 3: Multiplication as equal groups organised into an array

6 x 4 = 24 (means '6 equal groups of 4' or '4, 6 times')

Concrete:

Pegboard:

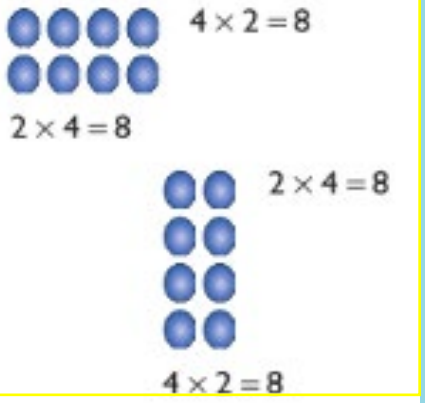
List the total of each row at the end (times tables / multiples)

Pictorial:

On squared paper:

●	●	●	●	4
●	●	●	●	8
●	●	●	●	12
●	●	●	●	16
●	●	●	●	20
●	●	●	●	24

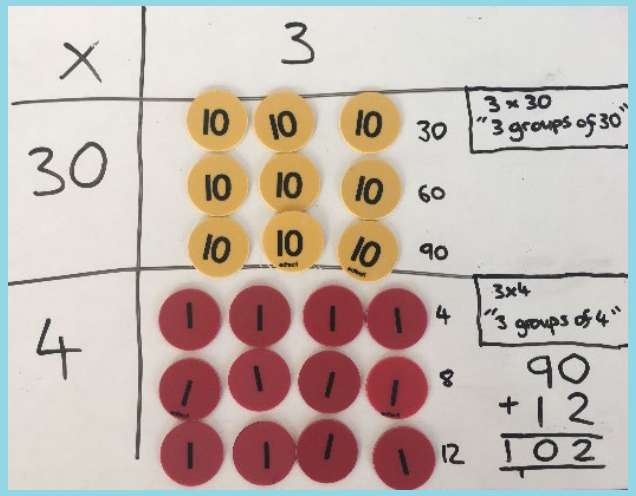
Also, recognise multiplication can be done in any order (commutative)



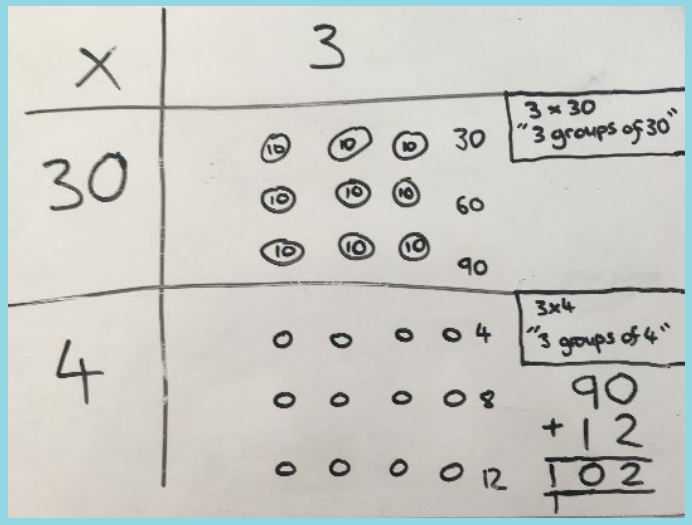
Stage 4 (also a mental method): Grid method (linked to calculating area)

34 x 3 = 102

Concrete:



Pictorial:



Abstract:

x	3	
30	90	
4	12	
+	102	
	1	

Steps (with language to use):

1. Draw a multiplication grid (TO x O).
2. Make an array, using counters, for the tens multiplied by the ones calculation first.
3. Write the product in the grid space.
4. Make an array, using counters, for the ones multiplied by the ones calculation next.
5. Write the product in the grid space.
6. Use the short column method of addition to add the two products together, to get the total.

Stage 5: Expanded long multiplication

Abstract:

56 x 27 = 1512

$$
\begin{array}{rl}
56 & \\
\times\ 27 & \\
\hline
42 & (7\times6) \\
350 & (7\times50) \\
120 & (20\times6) \\
+\ 1000 & (20\times50) \\
\hline
1512 & \\
\hline
1 &
\end{array}
$$

List each calculation by the side

Discussion/language to use:

Refer to the place value of the tens digits, so:

"7 x FIFTY, TWENTY x 6 , TWENTY x FIFTY"

If lacking confidence, can still use grid method for TO x TO, but it's inefficient

56 x 27 =

x	20	7	
50	1000	350	**1350**
6	120	42	**162**
		=	**1512**
			1

Stage 6: Long multiplication

Abstract:

56 x 127= 7112

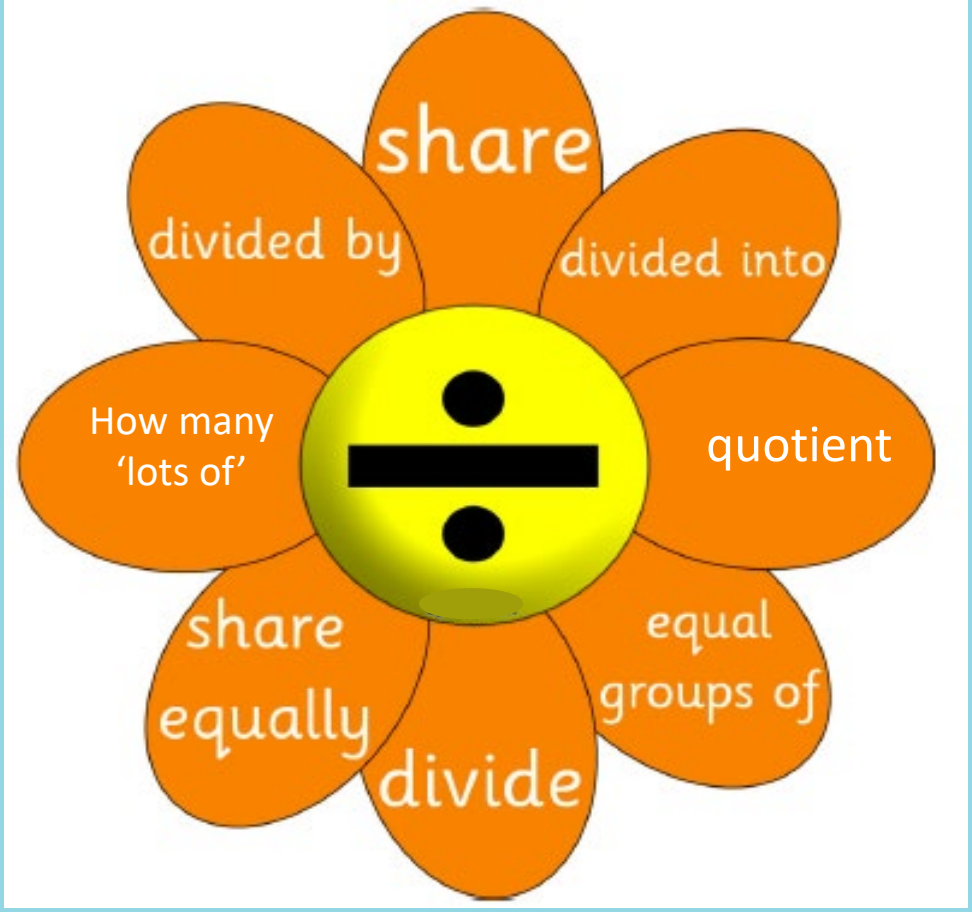
```
   56
x 127
  392
  3 4
 1120
  1 1
+5600
 7112
 1 1
```

Cross through exchanges after each row complete, so don't add up into final total by accident

Mark place holder zeros in a different colour, at first

Discussion/language to use:
Refer to the place value of the tens and hundreds digits, so:
"7 x FIFTY, TWENTY x 6 , TWENTY x FIFTY, etc."

Vocabulary for division operation:



Stage 1: Division as sharing equally into groups (put one 'thing' in each group at a time, before counting total in each group).

KEY VOCABULARY

halve

share, share equally

one each, two each, three each…

group in pairs, threes…

tens

equal groups of

divide

divided by

divided into

left, left over

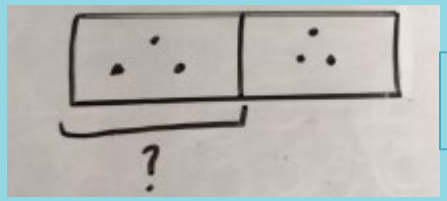
Concrete representation: e.g. everyday objects, counting bears, etc.

"6 shared equally between 3 is 2"

Pictorial representation: e.g. drawings, bar models, etc.

Sharing model

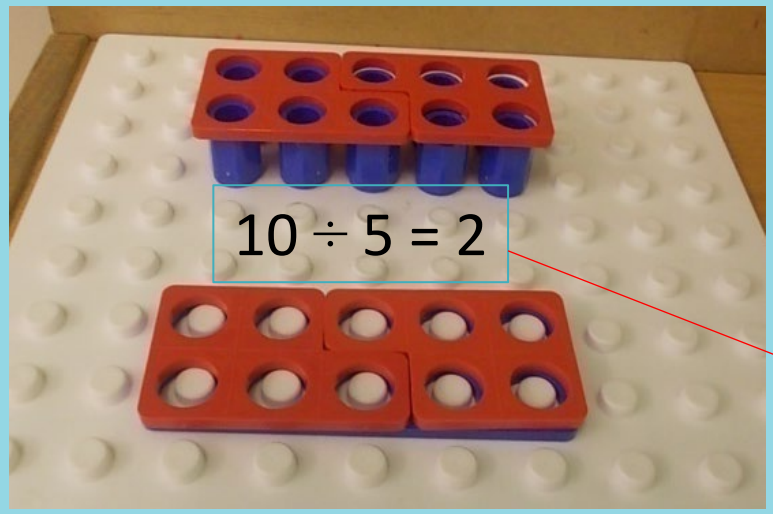
I have 10 sweets. I want to share them with my friend. How many will we have each?



"6 shared equally between 2 is 3"

Stage 2: Division as grouping equally

Concrete representation: e.g. Numicon, everyday objects, counting bears, etc.



"How many groups of 5 make 10?":

1. Make the whole
2. Group with same Numicon pieces
3. Count how many equal groups split into

Abstract representation/discussion alongside

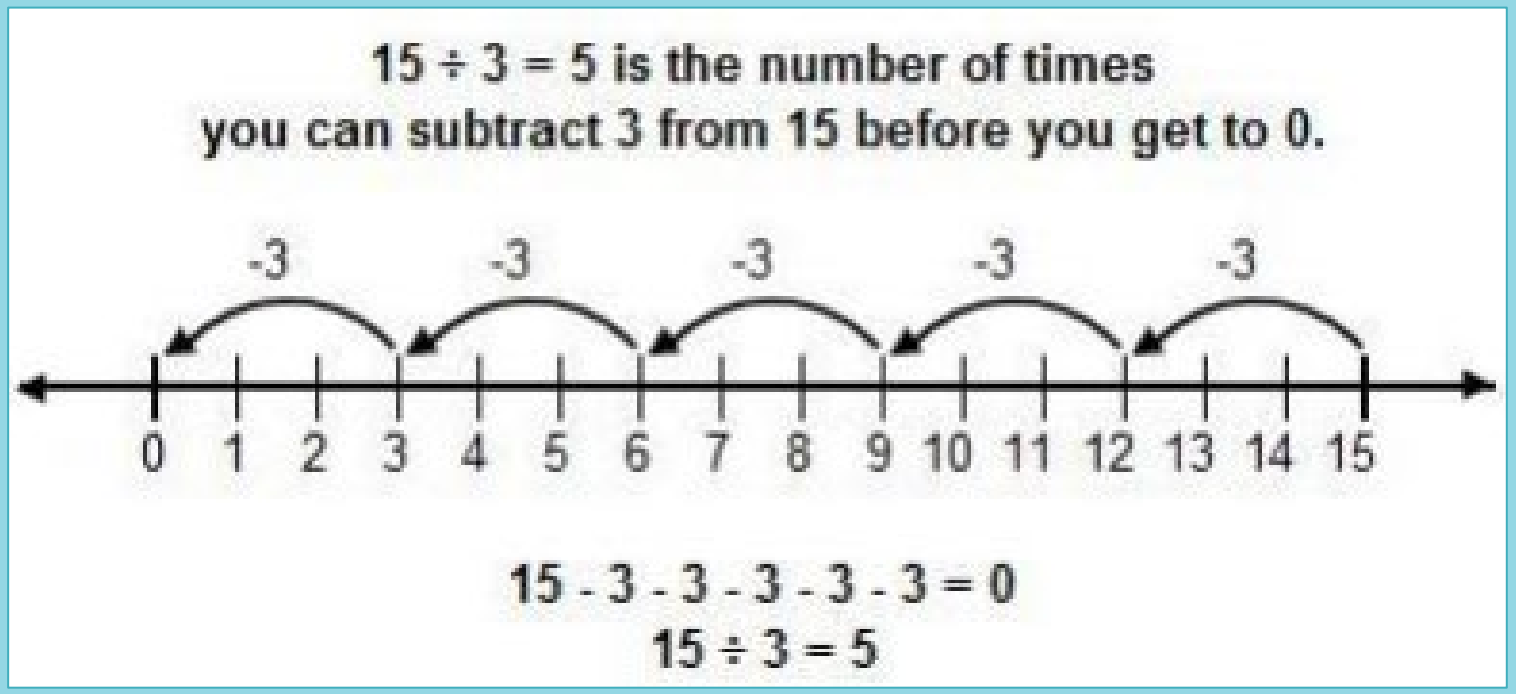
Pictorial representation: e.g. circling drawings, drawing groups, bar models, etc.

Grouping model
Mum has 6 socks. She grouped them into pairs – how many pairs did she make?

6 ÷ 3 = 2 (means 'how many groups of 3 make 6?')

Stage 3: Division as repeated subtraction (taking away equal groups on a filled number line)

15 ÷ 3 = 5 (means ‘how many groups of 3 make 15?’)



Stage 4: Division as grouping equally, into an array

$24 \div 4 = 6$ (means 'how many groups of 4 make 24?')

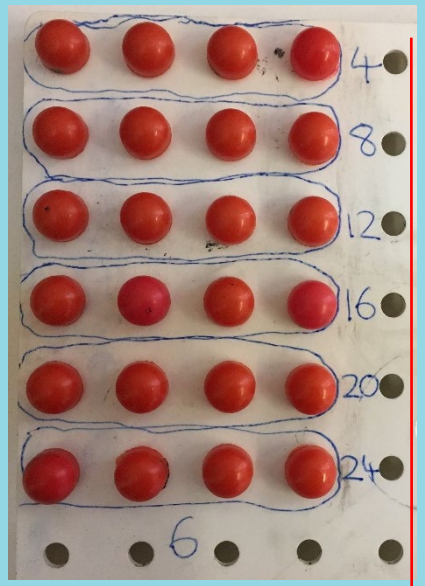
24 is the whole amount (the total altogether)

4 is the group size you are dividing into (the row)

6 is how many groups of 4 make 24

Concrete:

Pegboard:



List the total of each row at the end (times tables / multiples)

Pictorial:

On squared paper:

●	●	●	●	4
●	●	●	●	8
●	●	●	●	12
●	●	●	●	16
●	●	●	●	20
●	●	●	●	24
		6		

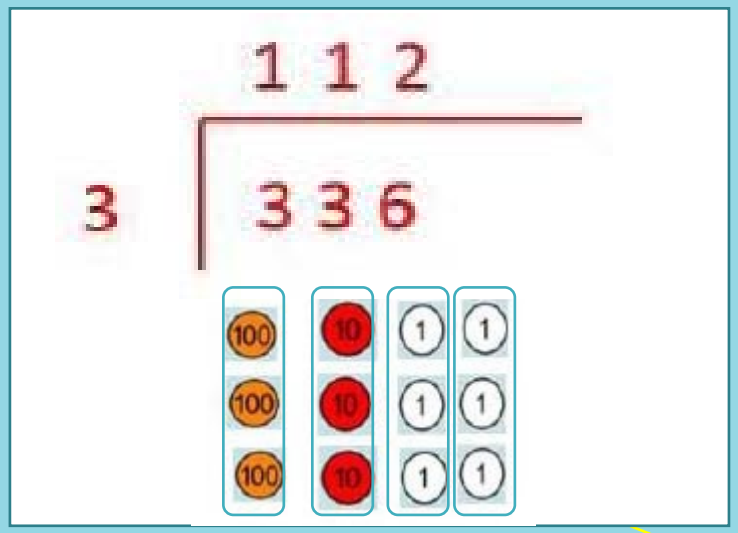
Circle and count how many groups altogether

Using multiplication facts is also a mental method

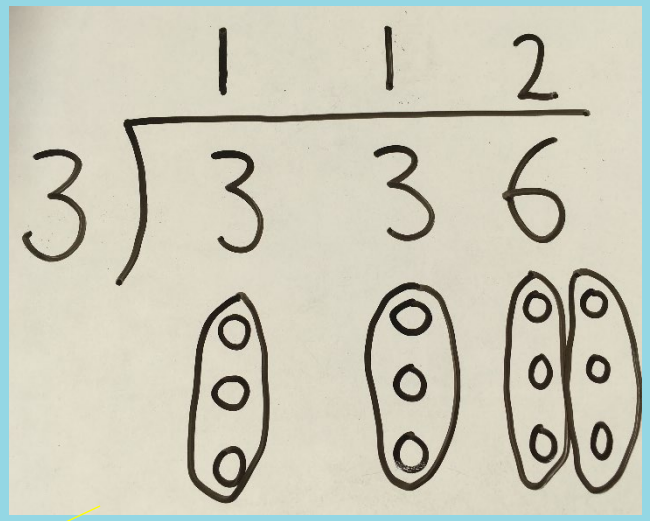
Stage 5: Short division, no remainders, with counters

336 ÷ 3 = 112

Concrete:



Pictorial:



Discussion/language to use:

"How many 'groups of' 3 hundreds are in the hundreds column?"

"How many groups of 3 tens are in the tens column?"

"How many groups of 3 ones are in the ones column?"

Stage 6: Short division with remainders

Abstract:

Stage 6a – remainders as fractions

$196 \div 6 = 32\frac{4}{6}$

$$6\overline{)1\,{}^{1}9^{1}6}\quad 0\ 3\ 2\ \text{r}4\ (\text{or}\ \tfrac{4}{6})$$

6
12
18
24
30
36
42

Stage 6a – remainders as fractions AND decimals

£385 ÷ 16 = £24.06

$$16\overline{)3\ 8^{3}\ 5^{6}.\ 0^{1}\ 0^{10}\ 0^{4}}\quad 0\,2\ 4\ .\ 0\ 6\ 2$$

(rounds to £24.06)

16
32
48
64
80
96
112

List multiples by the side to help keep track

Stage 7: Long division (taking away 'groups of' multiples of the number being divided by)

Abstract:

$196 \div 16 = 12.25$

$$
\begin{array}{r}
12\ \mathrm{r}\ \frac{4}{16} \\
16\overline{)\,196} \\
-\ 160 \quad (16 \times 10) \\
\hline
36 \\
-\ 32 \quad (16 \times 2) \\
\hline
4
\end{array}
$$

Solving Time Calculation Problems

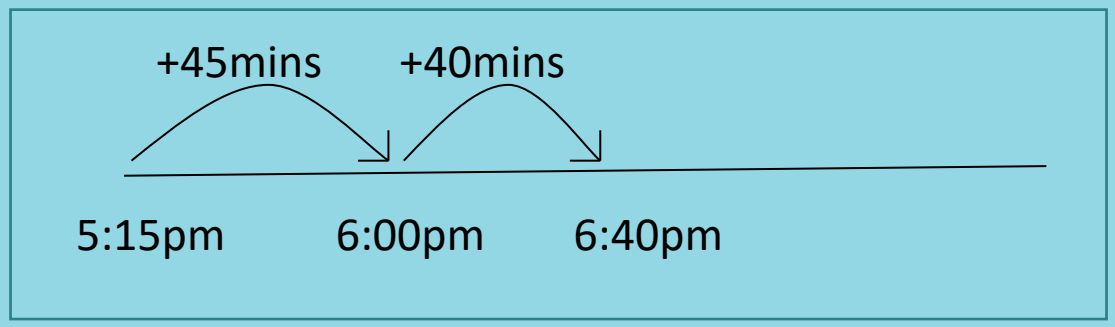
When solving problems which involve finding differences between times, or finding start/finish times when given a duration, it is useful for pupils to plot the calculation using an empty number line (no matter what year group).

As hours are out of 60, not 100 (base 10), normal formal written methods are not useful. Instead, if pupils count on in steps starting either to the next hour, or in hours, they will be more accurate with their calculations (see next page).

Example time problem (Year 4): *Imran's swimming lesson starts at 5:15pm. If his swimming lesson lasts 1 hour and 25 minutes, what time does it finish?*

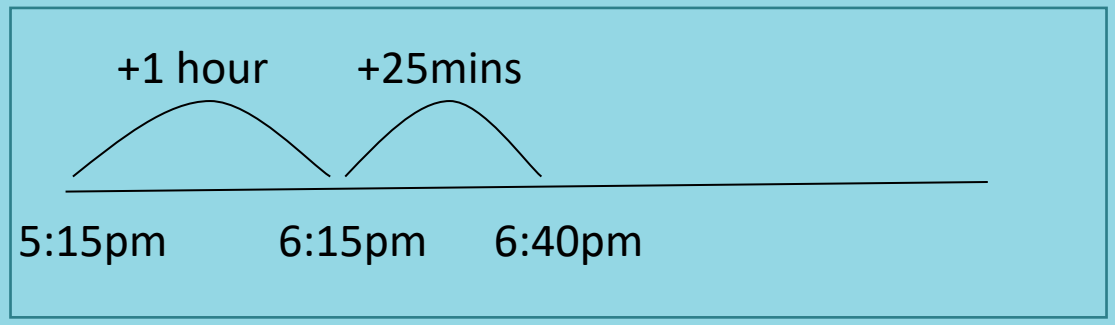
- First, convert the duration 1h 25mins to minutes (85), then either:

a) Count on to the next hour, then count on the remaining time before adding together the total steps:



OR

b) Count on in hours, then count on the remaining time before adding together the total steps:

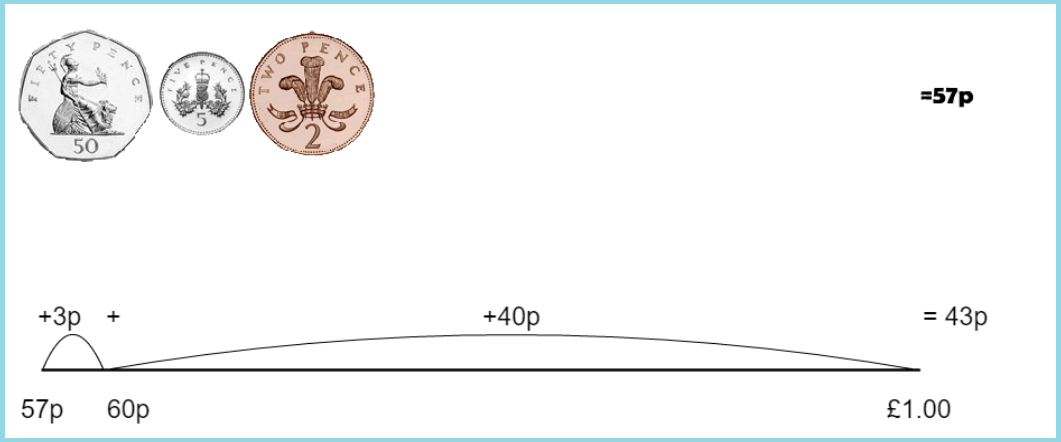


Solving Money Change Problems

Example problem (Year 3): *Mum bought three carrots costing 12p each and a tomato costing 7p. How much change did she get from £1.00?*

Use **Stage 4b (also a mental method):** Subtraction as 'finding the difference', by counting on, on an empty number line (see page 19)

43p + ? = £1.00



You can add marks to form a scale under it in any amount e.g. 1p, 2p, 5p, 10p etc.

Use knowledge of number bonds to 10 to "count on" in steps, until you get to the total spent.
If more than £1 is spent, convert to pennies first (e.g. £1.47 = 147p) and use related knowledge of number bonds to 100 to "count on" to 100p first.